Magnetized target fusion in cylindrical geometry

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Abstract

General ignition conditions for magnetized target fusion (MTF) in cylindrical geometry are formulated. To attain an MTF ignition state, the deuterium–tritium fuel must be compressed in the regime of self-sustained magnetized implosion (SSMI). We analyze the general conditions and optimal parameter values required for initiating such a regime, and demonstrate that the SSMI regime can already be realized in cylindrical implosions driven by ~ 100 kJ beams of fast ions. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Earlier, the concept of magnetized target fusion (MTF), based on a suggestion to introduce a magnetic field into inertial confinement fusion (ICF) targets, was advocated mainly for the case of spherical implosions [1–3]. Here we discuss basic physical issues of MTF in cylindrical geometry. Our interest in cylindrical geometry is primarily due to the prospects of using intense beams of fast ions in axial geometry to drive cylindrical implosions, and by a relative simplicity—compared with the spherical case—of creating strong initial magnetic fields in cylindrical targets.

The motivation for introducing a magnetic field into an ICF target is associated with a potential for strong reduction of the required driver power by reducing the fuel $\rho R$ value at ignition. One can show that, for a given ignition temperature, the driver power $W_{\text{dr}}$ required for ignition of a cylindrical fuel volume of radius $R$ at a density $\rho$ scales as $W_{\text{dr}} \propto (\rho R)^2/ C_r$, where $C_r = R_0/R$ is the radial convergence ratio of the implosion [4]. Hence, we seek to obtain ignition in the MTF mode at a minimum possible value of the fuel $\rho R$ parameter.

2. MTF ignition conditions

General conditions for thermonuclear ignition that must be fulfilled in a fuel volume with parameters $\rho$, $T$, $R$ in the assembled state are

(a) a positive net thermal balance, $dT/dt > 0$, in the stagnating fuel at maximum compression,
(b) a long enough confinement (dwell) time of the assembled state, \( t_{\text{con}} > t_{\text{ig}} \), for a thermonuclear flare to develop, and
(c) a long enough dwell time, \( t_{\text{Bdif}} > t_{\text{ig}} \), for the compressed magnetic field—when the latter is essential for the ignition process.

In the pure ICF mode, as is well known, conditions (a) and (b) lead to lower limits on the fuel temperature, \( T > 5 \) keV, and the \( \rho R \) parameter, \( \rho R > 0.2 \text{--} 0.5 \) g/cm\(^2\). Introduction of a transversal magnetic field \( B \), which inhibits the heat conduction and alpha particle losses in the radial direction, allows to reduce the \( \rho R \) threshold of ignition. As it is shown in Ref. [4], a substantial (by about a factor 10 or more) reduction of the ignition \( \rho R \) can be achieved only when the alpha Larmor radius becomes smaller than the fuel radius \( R \). In this case, the lower limit on the \( \rho R \) parameter dictated by condition (a) is replaced by a lower limit on the product \( BR \), and the ignition criterion becomes

\[
\begin{cases}
T = 7 \text{--} 10 \text{ keV}, \\
BR \geq (6.5 \text{--} 4.5) \times 10^5 \text{ G cm}.
\end{cases}
\]  

Once criterion (1) is fulfilled, the fuel \( \rho R \) is no longer limited by the requirement (a) of a positive thermal balance at stagnation. There remains, however, a limit imposed by condition (b) of inertial confinement. An effective way to enhance the inertial confinement of a low-\( \rho R \) fuel assembly is to surround it with a massive tamper composed of a high-Z material. Extensive one-dimensional (1-D) simulations of such configurations [5] have shown that the lower limit on the fuel \( \rho R \) at ignition is

\[
\rho R \geq 0.01 \text{--} 0.02 \text{ g/cm}^2.
\]  

Since the characteristic time

\[
t_{\text{Bdif}} = \frac{\pi}{c} \sigma_\perp R^2
\]  

for diffusive dissipation of the magnetic field due to the finite plasma conductivity \( \sigma_\perp \) is an additional independent timescale in the MTF ignition mode, one should demand that the inequality

\[
t_{\text{Bdif}} \geq t_{\text{ig}} = 3(n_B + n_T) T / Q_z
\]  

be fulfilled in addition to conditions (a) and (b) above. Here \( Q_z \) is the heating rate per unit volume due to the alpha energy deposition. Taking the Braginskii formula [6] for \( \sigma_\perp \), we find that Eq. (4) is equivalent to a lower limit on the fuel mass \( m = \pi \rho R^2 \) per unit cylinder length,

\[
m \geq \frac{1.85 \times 10^{-24} \bar{A} L_{\text{eq}}}{f_2 (\sigma v)_{DT} \sqrt{T_{\text{keV}}}} \text{[g/cm]}
\]

\[
= \begin{cases}
6 \times 10^{-4} \text{ mg/cm}, & T = 7 \text{ keV} \\
1.7 \times 10^{-4} \text{ mg/cm}, & T = 10 \text{ keV}.
\end{cases}
\]  

In Eq. (5) we used the values \( \bar{A} = 2.5, f_2 = 0.5, L_{\text{eq}} = 7 \) for the mean atomic mass \( \bar{A} \), the alpha energy deposition fraction \( f_2 \), and the Coulomb logarithm \( L_{\text{eq}} \); the CGS units are used in all practical formulae here and below. Note that limit (5) is sufficiently low and has no practical consequences for possible applications of MTF.

All the above criteria have been obtained under the assumption that the ignition fuel section has a sufficient \( z \) extension. With non-inhibited heat and alpha particle losses along the axial \( z \)-direction, one would expect approximately the same lower limit of 0.2\--0.5 g/cm\(^2\) on the fuel \( \rho z \) value as in the non-magnetized case.

3. Self-sustained regime of magnetized implosions

To reach the MTF ignition conditions formulated above, the hot core of the DT fuel must be imploded with minimal losses of the entropy and magnetic flux. Here and below we consider 1-D cylindrical target configurations with an axial magnetic field, initially created by external currents. In the ideal case of a quasi-adiabatic implosion with a fully frozen-in magnetic field, the field strength \( B \) and the DT temperature \( T \) would scale with the fuel density as

\[
B \propto \rho^{1/3}, \quad T \propto \rho^{2/3}.
\]  

This regime can be realized when the values of the following three basic dimensionless parameters

\[
x_e \equiv \omega_e \tau_e = 3.18 \times 10^{-7} \frac{\bar{A} B^3}{\rho L_{\text{eq}}} \text{[cm]}
\]
\[
\text{Pe} \equiv \frac{t_{ec}}{t_{im}} = \frac{n_e U R}{\kappa_{e,\perp}} = 3.15 \times 10^{-5} UR^{2} \frac{\rho L_{ei}}{AT_{keV}^{3/2}} \\
3.77 + 14.79x_e^2 + x_e^4 \\
11.92 + 4.664x_e^2
\]

(8)

\[
\text{Re}_m \equiv \frac{t_{\text{diff}}}{t_{im}} = \frac{2\pi}{c_e \sigma_{\perp}} UR = 0.019UR^{3/2} \frac{T_{keV}^{1/2}}{L_{ei}}
\]

(9)

all become significantly larger than 1. Here Pe and Re\textit{m} are the Péclet and the magnetic Reynolds numbers, \(\omega_e\) is the electron gyrofrequency, \(\tau_e\) is the electron collision time, \(t_{ec}\) is the timescale for the electron conduction cooling, \(t_{\text{diff}}\) is given in Eq. (3), \(t_{im} = R/U\) is the implosion timescale, \(U\) and \(R\) are the implosion velocity and the radius of the hot fuel region. In Eqs. (7)–(9) we used the Braginskii formulae [6] for \(\tau_e\), and for the transversal heat, \(\kappa_{e,\perp}\), and electrical, \(\sigma_{\perp}\), conductivities.

One readily verifies that, once the values \(x_e \gg 1\), \(\text{Pe} \gg 1\), \(\text{Re}_m \gg 1\) are established, they scale as \(x_e \propto \rho\), \(\text{Pe} \propto U\rho^{5/6}\), \(\text{Re}_m \propto U\rho^{1/2}\), and all three continue to grow in the course of implosion with a given implosion velocity \(U\). In this case we have a self-sustained regime of magnetized implosion (SSMI), whose quality in confining the entropy and magnetic flux only improves as the flow converges towards the axis. We assume that the required low level of the bremsstrahlung energy losses is ensured by a sufficiently low value of the fuel \(\rho R\). The SSMI regime is parametrically unstable in the sense that, once the values of either Pe or \(\text{Re}_m\) fall below unity, the imploding fuel begins to lose entropy and magnetic flux at an ever increasing rate.

The SSMI regime should not start necessarily with \(x_e > 1\) if we have initially \(x_e < 1\), then the Péclet number Pe—which now decreases with the density as \(\text{Pe} \propto U\rho^{-7/6}\)—should be large enough, so that it still remains above unity by the time when the growing \(x_e\) attains 1. As a result, we can combine the constraints on the initial values of \(x_e\) and Pe into a single condition

\[
x_e \text{Pe}|_{x_e=1} \equiv 1.18 \times 10^{-11} \frac{UR_0 B_0}{T_{keV}} \geq \mathcal{P} e_0
\]

(10)

where \(\mathcal{P} e_0\) is a fixed number on the order of or greater than 1. If we add now a lower limit on the magnetic Reynolds number, \(\text{Re}_m \geq \mathcal{P} e_0\) (where \(\mathcal{P} e_0\) is again a fixed number on the order of 1), we end up with the following two constraints on the initial state for the SSMI regime

\[
UR_0 \geq 8.5 \times 10^{10} \frac{\mathcal{P} e_0 T_{keV}}{B_0} \geq 1 \text{ cm}^2/\text{s}
\]

(11)

\[
UR_0 \geq 52 \mathcal{P} e_0 L_{ei} T_{keV}^{3/2} \geq 1 \text{ cm}^2/\text{s}.
\]

(12)

Note that all the quantities in Eqs. (11) and (12) correspond, strictly speaking, not to the initial target state but to the fuel state that we expect to implode subsequently along the quasi-adiabatic SSMI path. If, for example, a strong first shock is launched into the fuel, the values of \(T_0\) and \(B_0\) should be measured behind such a shock. Also, we ignore the difference between the initial radius \(R_0\) of the fuel and its radius by the time when the characteristic implosion velocity \(U\) is reached because the hydrodynamic timescale for the initial acceleration is again \(t_{im} = R_0/U\).

From inequalities (11) and (12) we find the optimum initial temperature

\[
T_{0,\text{opt}} = 20 \text{ eV} \left( \frac{B_0}{10^5 \text{ G}} \right)^{2/5} \left( \frac{\mathcal{P} e_0 L_{ei}}{\mathcal{P} e_0} \right)^{2/5}
\]

(13)

and the absolute lower bound on the product \(UR_0\),

\[
UR_0 \geq 1.8 \times 10^4 \left( \frac{10^5 \text{ G}}{B_0} \right)^{3/5} \mathcal{P} e_0^{3/5} \left( \mathcal{P} e_0 L_{ei} \right)^{2/5} \text{[cm}^2/\text{s]}
\]

(14)

required for the regime of self-sustained magnetized implosion.

4. Magnetized implosions driven by a 100 kJ ion beam

In the near future, ion beams with total beam energies of \(E_b = 1–100\) kJ and a specific energy deposition level of \(\varepsilon \approx 100\) kJ/g in high-Z materials are planned to be available at ITEP (Moscow, the TWAC project) [7] and GSI (Darmstadt) [8,9]. Having ion ranges well in excess of the focal spot radius \(r_{\text{foc}} \lesssim 1 \text{ mm}, such beams can be used for implosion experiments in cylindrical geometry, as it is shown schematically in Fig. 1. If only a part of the range is used, the non-uniformity of the ion
energy deposition along its trajectory is not strong (\(\lesssim 5\%\) over the first 50–60\% of the range of \(\pm 1\) GeV/u ions—which amounts to \(\pm 10\) mm in solid gold) and can, for example, be compensated by an appropriate shaping of the cold pusher thickness. Peak pressures of up to 100 Mbar and higher can be reached in a converging cylindrical flow when an annular region \((r_{\text{foc}} < r < r_{\text{foc}})\) of a high-Z liner is heated by the ion beam and accelerates a cold pusher (initially at \(R_0 < r < r_{\text{foc}}\)) towards the cylindrical axis. To take precautions against the Rayleigh-Taylor instability during the acceleration phase, we assume a not too high \((C_{r4} < 10)\) aspect ratio for the cold pusher. More precisely, we used the following proportions in the 1-D simulations described below: \(R_0 = 0.55r_{\text{foc}}, r_{\text{foc0}} = 0.6r_{\text{foc}}, R_0 = 1.5r_{\text{foc}}.\)

If, now, the deuterium gas in the central cavity is adequately preheated and magnetized, one can expect a significant enhancement of the peak deuterium temperature and the neutron yield when the SSMI regime is reached. For not too long ion pulses, the principal condition (14) for this regime transforms into a lower limit on the parameter

\[
\frac{E_h}{\langle \rho l \rangle_i} \geq 13 \left( \frac{10^5 \text{ G}}{B_0} \right)^{6/5} \left[ \frac{\text{kJ}}{\text{g/cm}^2} \right]
\]

where \(\langle \rho l \rangle_i\) is the ion range along the cylindrical axis. Limit (15) is still far from being reached by a 20 kJ option for the future GSI beam [9] with realistic values of \(B_0\), but is marginally approached by the TWAC beam parameters [7] for \(B_0 \approx 3 \times 10^5\) G. The results of 1-D magneto-hydrodynamic simulations for the TWAC beam \((\varepsilon = 100\text{ kJ/g}, r_{\text{foc}} = 0.8\text{ mm}, \text{pulse duration 100 ns})\) with the DEIRA code [10], presented in Fig. 2, do indeed demonstrate that a noticeable
(by about a factor 1.6) enhancement of the peak deuterium temperature can be expected in magnetized implosions with the initial axial field of $B_0 = 3 \times 10^5$ G. The optimal initial deuterium parameters are close to $\rho_0 = 10^{-4}$ g/cm$^3$, $T_0 = 20$ eV. The effect of fuel magnetization on the total number of thermonuclear neutrons (Fig. 2b) is less pronounced than one would expect from the central temperature enhancement shown in Fig. 2a because of the magnetic flux and entropy losses from the fuel periphery, which contains most of the fuel mass.

References